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*FINAL REPORT FOR THE GRANT*

**A FUNDAMENTAL STUDY OF GAS AND VAPOR  
BUBBLE DYNAMICS IN MICRO-CHANNELS**

AFOSR GRANT NO. F49620-96-1-0386

A. Prosperetti and W.N. Sharpe  
Department of Mechanical Engineering, The Johns Hopkins University  
Baltimore, MD 21218

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# 1 Introduction

The aim of this project was to carry out a fundamental study of the basic physics underlying the applications of gas and vapor bubbles in heat transfer systems, pumps, actuators, and other small-scale systems. Since these applications require a detailed understanding of the physics of bubbles in highly confined spaces, such as micro-channels, a large fraction of the effort has been devoted to this type of problems.

The physical situation differs greatly depending on whether the bubble considered contains predominantly vapor of the surrounding liquid (as when the bubble is initiated by heating), or a permanent gas. Accordingly, we distinguish between the two cases in the following.

In the course of the study some new pumping principles have been discovered and a patent application filed on July 6, 1999. A response to the Patent Officer comments has been submitted in May 2000. Some of the claims of the original patent have been approved; for some of those denied, an appeal has been filed.

This work has had ample resonance in the media being mentioned in over 30 newspaper and magazine articles (including the Los Angeles Times, Business Week, and Popular Science), web-based publications (e.g. ABCNews, Academic Universe, Technical Insight Alert), and the professional press (e.g. Process Engineering, Water Environment & Technology, R & D Magazine, Engineering Times, Medical Devices & Diagnostic Industry).

Of course, of greater importance are the scientific publications which are detailed in the attached list.

## Publications

This grant has resulted in the following publications:

1. Oğuz, H.N. and Prosperetti, A. The Natural Frequency of Oscillation of Gas Bubbles in Tubes. *J. Acoust. Soc. Am.* **103**, 3301-3308, 1998.
2. Chen, X.M. and Prosperetti, A. Thermal Processes in the Oscillations of Gas Bubbles in Tubes. *J. Acoust. Soc. Am.* **104**, 1389-1398, 1998.
3. Geng, X., Yuan, H., Oğuz, H.N., and Prosperetti, A., The Oscillations of Gas Bubbles in Tubes: Experimental Results, *J. Acoust. Soc. Am.* **106**, 674-681, 1999.
4. Yuan, H., Oğuz, H.N., and Prosperetti, A., Growth and collapse of a vapor bubble in a small tube, *Int. J. Heat Mass Transfer* **42**, 3643-3657, 1999.
5. Yuan, H. and Prosperetti, A. The pumping effect of growing and collapsing bubbles in a tube, *J. Micromech. Microeng.*, **9**, 402-413, 1999
6. Ory, E., Yuan, H., Prosperetti, A., Popinet, S., and Zaleski, S. Growth and collapse of a vapor bubble in a narrow tube, *Phys. Fluids* **12**, 1268-1277, 2000
7. Geng, X., Yuan, H., Oğuz, H.N. and Prosperetti, A. Single-Bubble-Based Micropump, *Sensors and Actuators*, submitted

A copy of the first page of each is attached.

Reports on aspects of the work were also presented at several conferences:

1. H. Yuan, X. Chen, H.N. Oğuz, and A. Prosperetti, Dynamics of gas and vapor bubbles in microchannels, 50th Annual Meeting of the Division of Fluid Mechanics of the American Physical Society, San Francisco Ca November 23-25 1997; abstract published in the *Bull. Am. Phys. Soc.* **42** (11), p. 2194 (1997).

2. H. Yuan and A. Prosperetti, Bubble-based pump for small channels, 136th Meeting of the Acoustical Society of America, Norfolk VA 12-16 October 1998; abstract published in *The J. Acoust. Soc. Am.* **104**, p. 1770 (1998).
3. X. Geng, H. Yuan and A. Prosperetti, The oscillations of a gas bubble in a tube, 136th Meeting of the Acoustical Society of America, Norfolk VA 12-16 October 1998; abstract published in *The J. Acoust. Soc. Am.* **104**, p. 1771 (1998).
4. H. Yuan and A. Prosperetti, Bubble-based pump for small channels, 51st Annual Meeting of the Division of Fluid Mechanics of the American Physical Society, Philadelphia PA November 22-24 1998; abstract published in the *Bull. Am. Phys. Soc.* **43** (9), p. 1985 (1998).
5. X. Geng, H. Yuan, H.N. Oğuz, and A. Prosperetti, The oscillations of a gas bubble in a tube, 51st Annual Meeting of the Division of Fluid Mechanics of the American Physical Society, Philadelphia PA November 22-24 1998; abstract published in the *Bull. Am. Phys. Soc.* **43** (9), p. 1966 (1998).
6. H. Yuan, E. Ory, and A. Prosperetti, A micro-pump powered by a single vapor bubble, 52nd Annual Meeting of the Division of Fluid Mechanics of the American Physical Society, New Orleans LA November 21-23 1999; abstract published in the *Bull. Am. Phys. Soc.* **44** (8), p. 97 (1999).

In addition to the preceding papers and conference presentations, periodic progress reports have been published in the Proceedings of the DARPA MEMS PI Meetings held during the period of the grant. Furthermore, partial descriptions of the work were given in several invited seminars presented by the authors on several occasions.

## Personnel

The following persons have been supported totally or in part under this grant; all are or have been affiliated with the Department of Mechanical Engineering of the Johns Hopkins University:

1. Dr. Andrea Prosperetti, Professor, PI
2. Dr. William N. Sharpe, Professor, Co-PI
3. Dr. Hasan N. Oğuz, Associate Research Professor
4. Dr. He Yuan, Research Scientist
5. Dr. Emmanuel Ory, Post-Doctoral Fellow
6. Ms. Xuemei Chen, Graduate Student
7. Mr. Xu Geng, Graduate Student

## Unobligated Funds

No unobligated funds existed at the end of the period of performance.

## 2 Vapor Bubbles

A central ‘building block’ of the situations studied is the growth of a vapor bubble in a channel under the action of an impulse of heat. We have devoted a considerable effort to this problem. After a summary of this work, we describe two new pumping principles.

### 2.1 Growth and collapse of a vapor bubble in a small channel

The first study, published in Yuan et al. (1999), considers the fate of a small spherical bubble surrounded by a small region of hot liquid in a cylindrical channel (Fig. 1). The intent is to model the behavior of a bubble nucleus after the brief application of thermal energy.

The initial bubble radius is  $a(0)$  and the initial temperature distribution surrounding it is given (in polar coordinates) by

$$T(r, 0) - T_\infty = \Delta T \exp \left[ -\frac{r - a(0)}{\lambda} \right], \quad (1)$$

where  $\Delta T$  and  $\lambda$  are prescribed (the former of the order of 200-300 °C, the latter of the order of a few micrometers). Several combinations of these parameters have been studied, as well as several tube radii  $R_T$  (of the order of tens of micrometers) and lengths  $L$  (of the order of millimeters); some examples of the results are shown in Figs. 2 to 5.

In obtaining these results, we have assumed cylindrical symmetry stipulating that the bubble is generated and remains on the tube axis. We took the bubble to be a sphere from inception until its radius is nearly equal to the tube radius. Further growth is modelled by inserting a cylindrical volume between two hemispheres as shown in Fig. 1. When the bubble collapses, we switch back from the elongated to the spherical model when the height of the cylindrical volume becomes zero.

A limitation of the model was that inviscid flow was assumed and that the thermal aspects were also simplified by using a low-order collocation procedure. These approximations were made

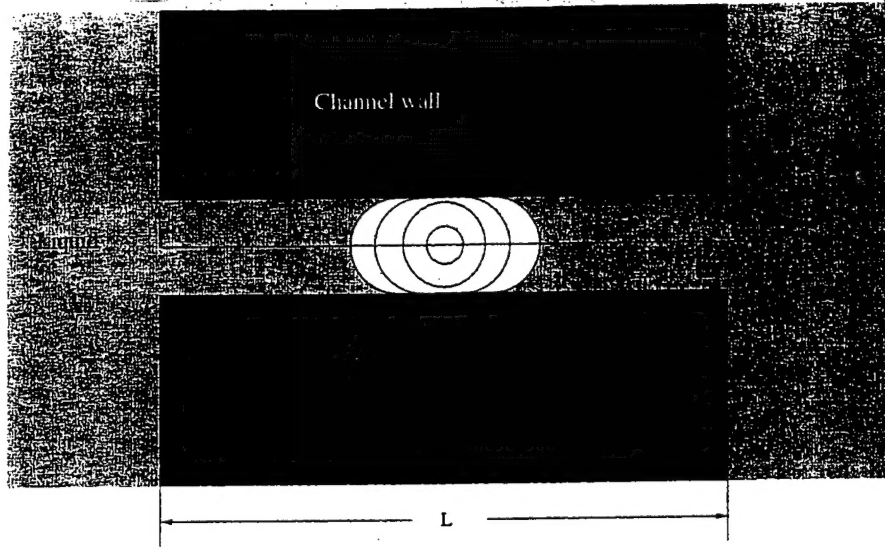


Figure 1: Sketch of the model studied in Yuan et al. (1999). When the bubble is sufficiently small, it is approximated as a sphere. After it has grown to nearly occlude the tube, it is approximated as a cylinder with two hemispherical caps.

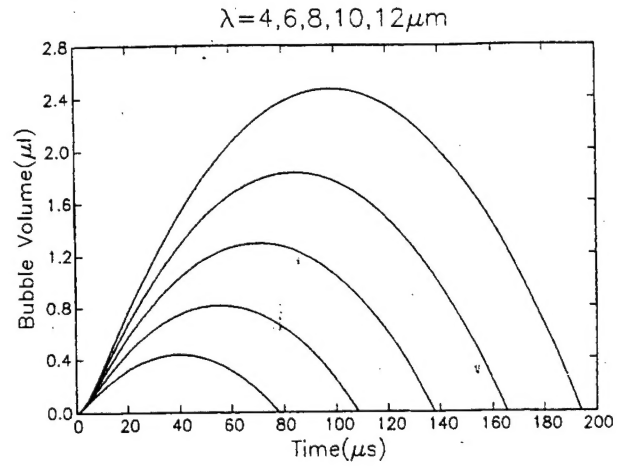
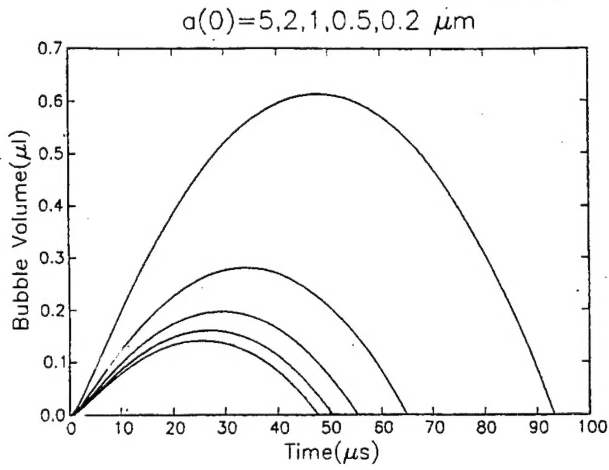


Figure 2: (left) Bubble volume versus time for initial bubble radii  $a(0) = 0.2, 0.5, 1, 2, \text{ and } 5 \mu\text{m}$  (in ascending order) for a given fixed amount of initial liquid thermal energy. Channel length and radius are  $5 \text{ mm}$  and  $50 \mu\text{m}$  respectively, with  $L/R_T = 100$ . The initial bubble radius is  $5 \mu\text{m}$ , so that  $R_T/a(0) = 10$ . The initial temperature distribution is given by (1) with  $\Delta T = 250 \text{ K}$ ,  $T_\infty = 300 \text{ K}$ ,  $\lambda = 5 \mu\text{m}$ . The liquid is water.

Figure 3: (right) Bubble volume versus time for different initial liquid temperature distributions (1) with  $\lambda = 4, 6, 8, 10, 12 \mu\text{m}$  (in ascending order). All other conditions as in Fig. 2.

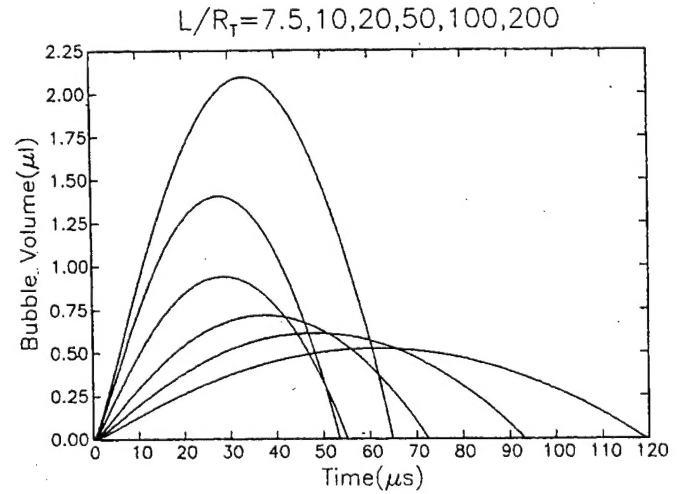
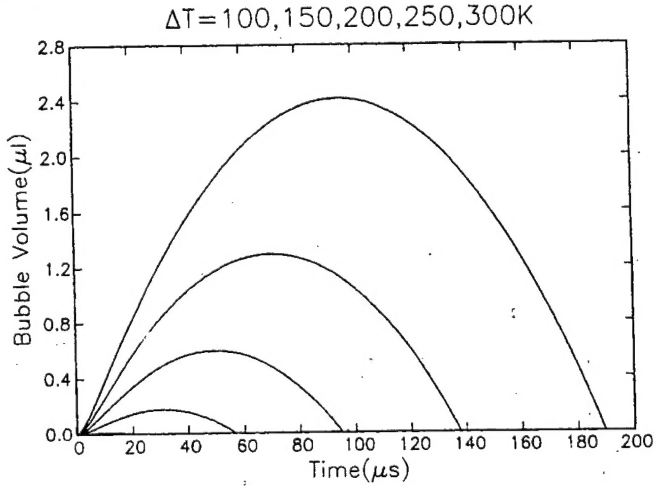


Figure 4: (left) Bubble volume versus time for different initial liquid superheats,  $\Delta T = 100, 150, 200, 250, 300 \text{ K}$  (in ascending order). All other conditions as in Fig. 2.

Figure 5: (right) Bubble volume versus time for different channel aspect ratios  $L/R_T = 7.5, 10, 20, 50, 100, 200$  (in descending order). In all cases, the bubble is at the center of the channel. All other conditions as in Fig. 2.

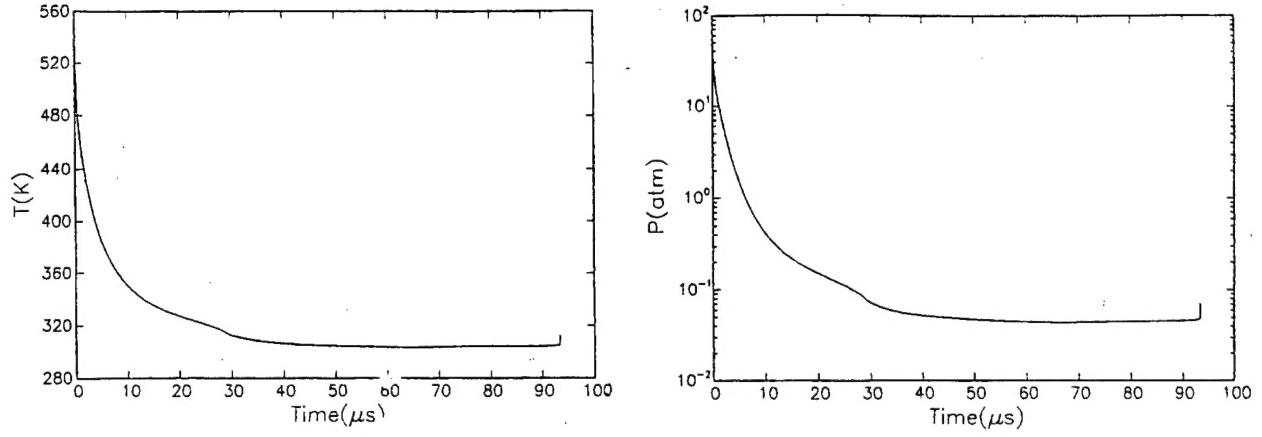


Figure 6: (left) Bubble internal pressure as a function of time: channel length and radius 5 mm and 50  $\mu m$  respectively; initial bubble radius 5  $\mu m$ ; initial temperature distribution given by (1) with  $\Delta T = 250$  K,  $T_\infty = 300$  K,  $\lambda = 5 \mu m$ ; the liquid is water.

Figure 7: (right) Bubble surface temperature as a function of time for the same conditions as in the previous figure.

necessary by the fact that, currently, no numerical method exists to deal simultaneously with the heat transfer and fluid dynamics of a situation of this type.

Nevertheless, an interesting result of these calculations – which has permitted us to perform better calculation later on – is shown in Fig. 6, where the internal bubble pressure (left) and surface temperature (right) are plotted as functions of time.

The significant feature here is that, after a brief initial transient, the pressure falls to a negligible value and the surface temperature drops to essentially the ambient value. This observation has enabled us to introduce a useful approximation in subsequent papers (Yuan & Prosperetti 1999; Ory et al. 2000): we approximate the pressure inside the bubble by a step function:

$$p(t) = \begin{cases} p_0 + \Delta p & 0 \leq t < \tau \\ p_0 & \tau \leq t \end{cases}, \quad (2)$$

in which  $p_0$  is the saturation pressure corresponding to the undisturbed liquid temperature and  $\Delta p$  an initial overpressure simulating the effect of a brief heating pulse.

With (2), the thermal problem is eliminated and one can focus on doing a better job on the fluid mechanic aspect of the problem. This we have done in Ory et al. (2000) where the full Navier-Stokes equations were solved by a volume-of-fluid method, still with the assumption of axial symmetry. While the paper contains a parametric study of the phenomenon for a wide range of different conditions, we focus here specifically on a very interesting effect that lies at the root of the first novel pumping principle that we have discovered. This effect is illustrated in Fig. 8, in which the upper panel shows the bubble contours during growth and the lower panel during collapse.

Here the liquid is water at 20 °C, the tube 2 mm long tube, with a diameter of 62.5  $\mu m$ ; the undisturbed pressure is 1 atm, the undisturbed saturation pressure  $p_0 = 0.02$  bars, the overpressure  $\Delta p$  is 50 bars, and the overpressure duration is  $\tau = 2 \mu s$ ; successive bubble shapes are separated by 2.6 ms. The actual computational domain extends between 0 and 2000  $\mu m$ , and therefore considerably beyond the left and right frame boundaries of the figures. The initial bubble nucleus is positioned 3/8 of the tube length away from the left end of the tube.

It is evident from these figures that the axial position where the collapse is completed – 859  $\mu m$  from the tube left end – is different from that of the original bubble nucleus – 750  $\mu m$  away from



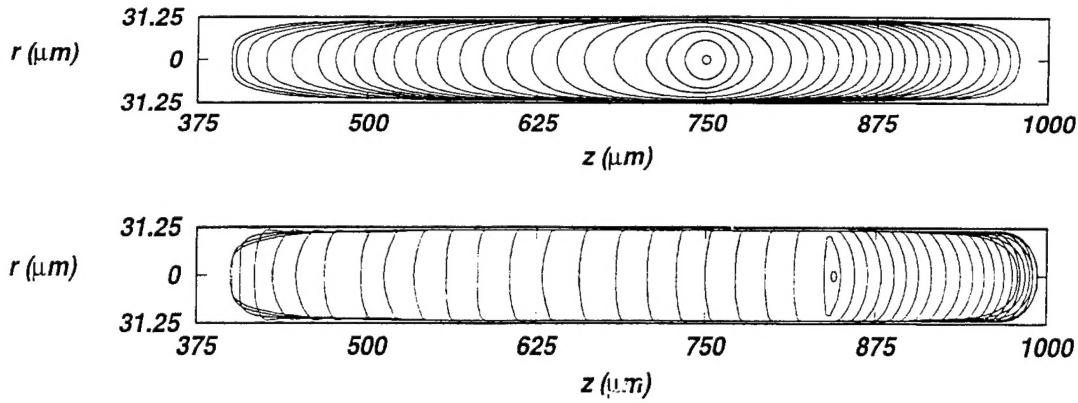


Figure 8: The action of a bubble actuated micropump – see text.

the tube left end. Thus, the bubble moves  $109 \mu\text{m}$  (i.e., 1.74 tube diameters) away from the closest end of the tube. Since the bubble essentially occludes the tube during most of its life, this result suggests that the growth and collapse cycle is capable of imparting a net displacement to the liquid. Thus, by generating successive bubbles in a periodic fashion, one would be able to pump the liquid from one reservoir to the other one along the tube.

## 2.2 A bubble-based micropump

We have verified this prediction experimentally in a device sketched in Fig. 9: a microchannel (diameter  $100\text{--}200 \mu\text{m}$ ) is molded onto a plexiglass plate and covered with another plate. The liquid is salt water and the localized heating is provided by two current-carrying wires spaced by a few hundred  $\mu\text{m}$ . A sequence of images of the bubble generated in these conditions is shown in Fig. 10. Here the left tube exit is closer to the bubble than the right one, and a phenomenon similar to that of Fig. 8 is apparent. (The two dark bands are the current-carrying wires).

The measured pressure head vs. flow rate developed by several pumps of a similar construction is shown in Fig. 11. In all cases the channel diameter is of the order of  $100\text{--}200 \mu\text{m}$ , and the channel length 26 mm. The large head developed by these tiny devices is quite remarkable.

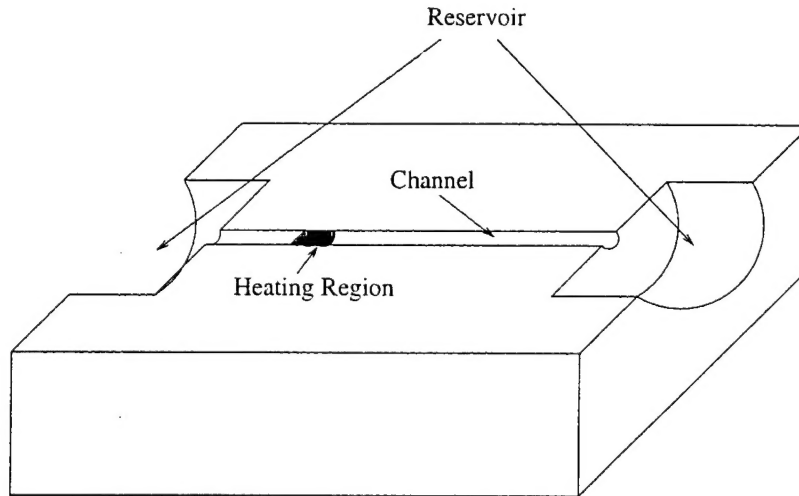


Figure 9: Cutaway sketch of the first new pump concept.

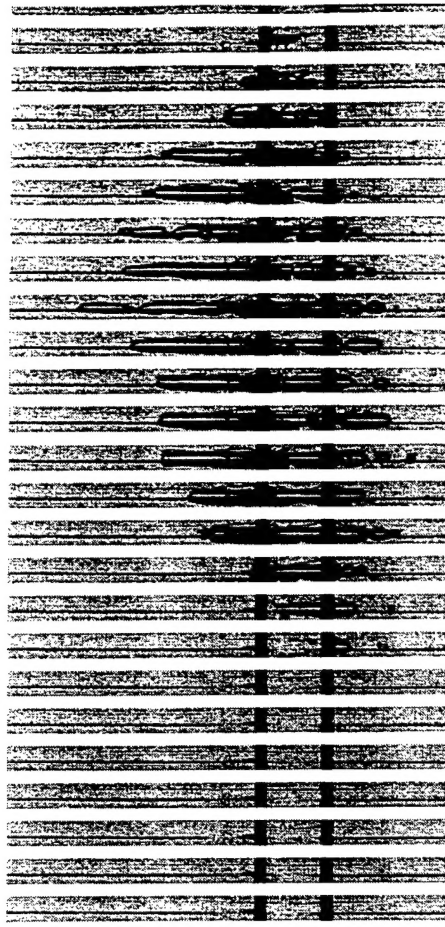


Figure 10: Sequence of bubble growth and collapse in the pump sketched in the previous figure.

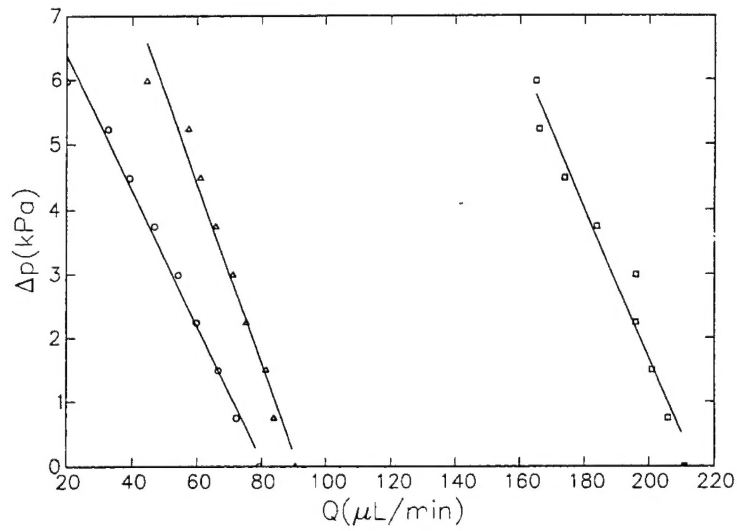


Figure 11: Measured pressure head vs. flow rate developed by several pumps of the type sketched in Fig. 9; in all cases the channel diameter is of the order of 100-200  $\mu\text{m}$ , and the channel length 26 mm.

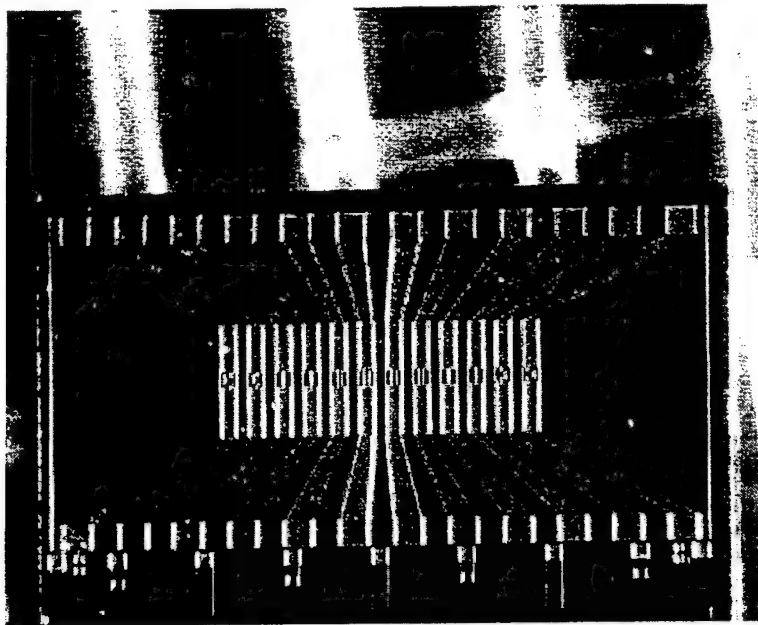


Figure 12: Microheaters deposited on a silicon substrate. When covered by a suitably shaped cover, this will be a multi-heater analogue of the device shown in Fig. 9.

In Yuan & Prosperetti this idea is extended to the case of several heaters arranged along the channel. The theoretical estimate shows that an even greater pumping effect can be achieved in this way, but no experiments have been conducted yet on this arrangement.

The ultimate embodiment of the idea requires the microfabrication of heaters deposited on silicon; Fig. 12 is an example of a first prototype recently completed. The fashioning of a channel will require covering the row of heaters with a suitably shaped plate; this is a problem that we are currently working on.

Work on this project is continuing under sponsorship of NSF.

### 2.3 Another bubble-based micropump

We have developed a completely different concept for another bubble-based micropump. The system is sketched in Fig. 12: a conical opening connects two electrically conducting tubes separated by a throat. When the system is filled with salt water and a potential difference established between the tubes, the current lines are 'squeezed' at the throat and cause intense heating capable of producing a bubble; when the current stops, the bubble condenses.

A photographic sequence of this process is shown in Fig. 14, where the indicated times are measured from the beginning of the heating; the current is turned off at  $t = 100$  ms. Again it is seen that the bubble is generated near the bottom of the conical passage, but condenses higher up: a volume of liquid of the order of the volume of the cone is thus pushed up at every growth/collapse cycle.

The mechanism of operation of this device may be described as follows. Surface tension causes a pressure difference between the bubble interior and the surrounding liquid. Due to the geometry of the device, the radius of curvature  $R_1$  of the lower part of the bubble is smaller than the radius of curvature  $R_2$  of the upper part. With reference, for example, to the points on the axis of symmetry, we can write

$$p_v - p_1 = \frac{2\sigma}{R_1}, \quad p_v - p_2 = \frac{2\sigma}{R_2} \quad (3)$$

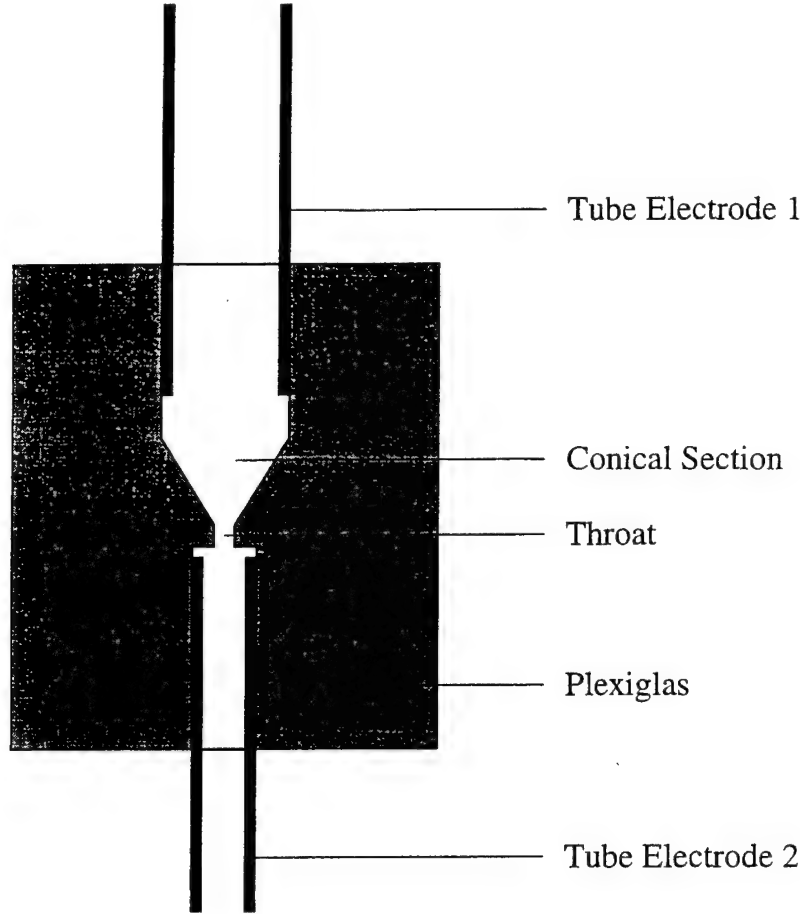


Figure 13: The second new pump concept.

where  $\sigma$  is the surface tension coefficient and  $p_v$  is the vapor pressure in the bubble. It is well known from the boiling literature that the liquid surface temperature inside a vapor bubble is essentially uniform due to the rapidity with which evaporation and condensation processes wipe out any temperature difference. Thus, the value of the surface tension coefficient can be assumed to be constant over the bubble surface. Furthermore, as long as the evaporation/condensation fluxes are well below the speed of sound in the vapor – a condition that is amply satisfied here – thermodynamic equilibrium can be assumed at the liquid-vapor interface, so that the vapor pressure  $p_v$  is also uniform inside the bubble. Thus, subtracting, we find from (3):

$$p_1 - p_2 = -2\sigma \left( \frac{1}{R_1} - \frac{1}{R_2} \right) < 0. \quad (4)$$

This relation shows that, in order to maintain equilibrium, the liquid pressure in the smaller tube upstream of the the throat must be kept below the liquid pressure in the wider tube. In other words, there is a pressure increase in traversing the device from the narrow to the wide tube: unless this pressure difference is maintained, there will therefore be a net flow upward.

The principle of operation of this pump is based on surface tension, while that of the previous one is based on inertia. Thus, while the effectiveness of the previous pump deteriorates going to small scales, that of the present one increases.

With the limited microfabrication facilities at our disposal, so far we have only been able to

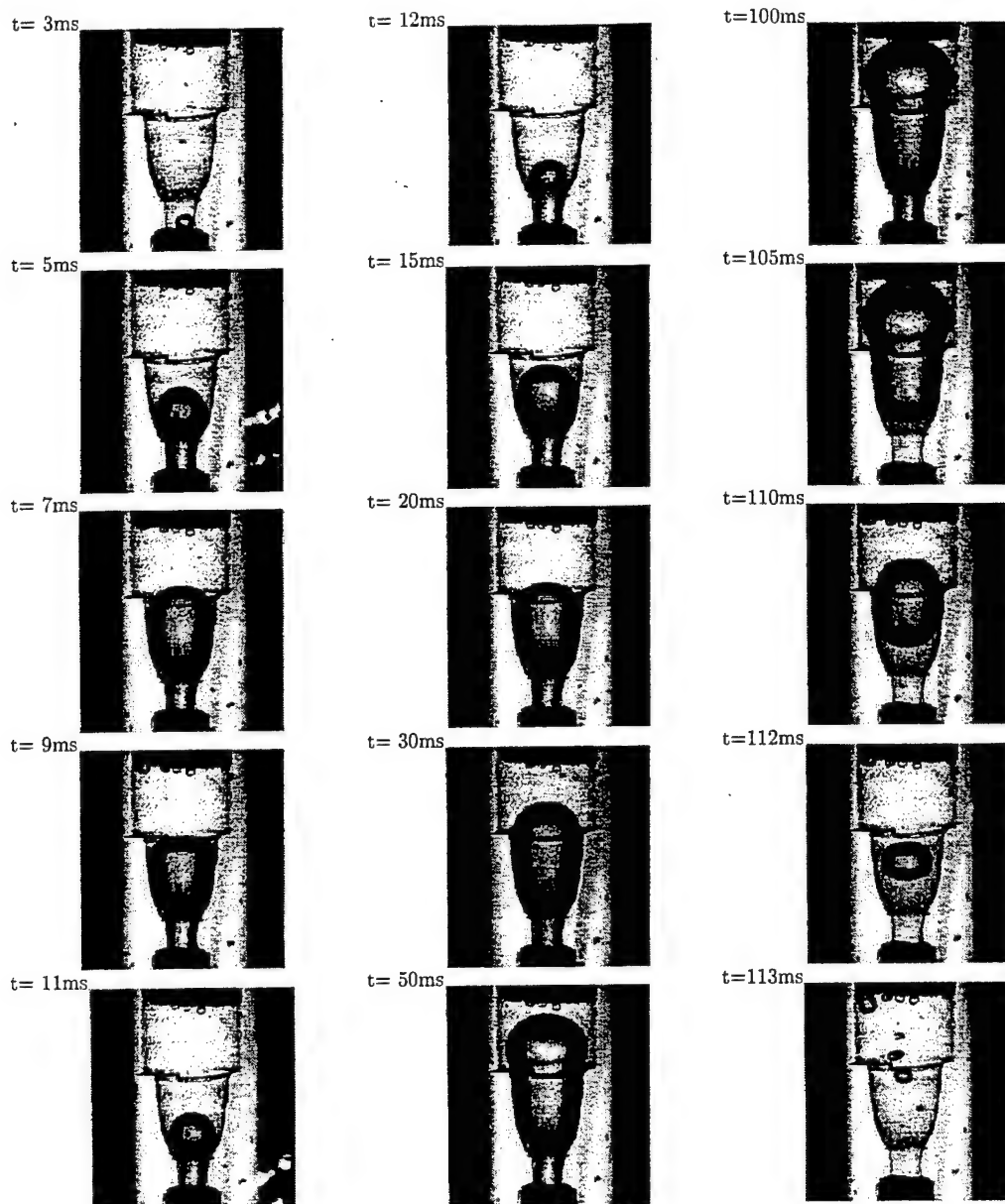


Figure 14: Bubble growth and collapse sequence for the pump of the previous figure.

manufacture a rather large embodiment of this idea (tube diameter of the order of  $500\ \mu\text{m}$ ), and the head developed is therefore much less than for the previous pump. An example showing the head at zero flow rate as a function of the repetition rate of the applied current  $T$  is shown in Fig. 15.

A preliminary description of this work is presented in Geng et al. (2001).

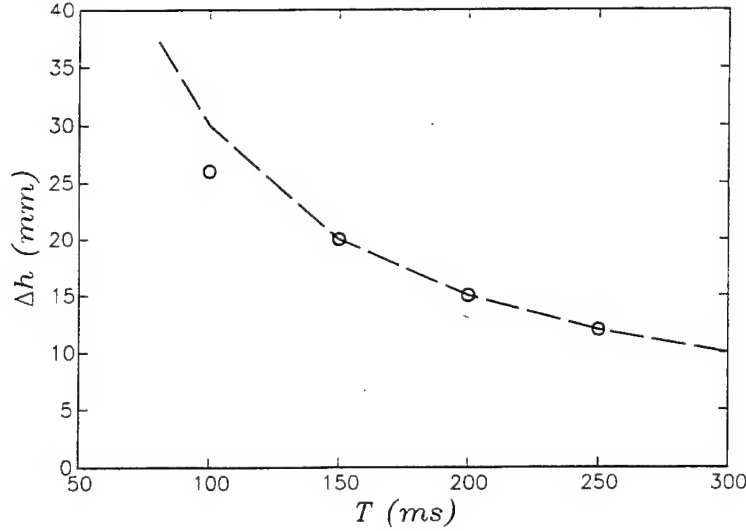


Figure 15: Zero-flow-rate head developed by the pump of the previous two figures vs. the repetition rate of the current  $T$ ; the current is applied for 100 ms in each cycle.

### 3 Gas Bubbles

Gas bubbles offer the interesting possibility of being driven remotely by a sound field. In this way, a gas-bubble-powered device would not need to carry its own energy source, but could be activated by an external sonic or ultrasonic field. In particular, sound can propagate harmlessly (or very nearly so) through biological tissue, which opens the interesting possibility of biomedical applications of such devices.

For any such application, the most significant features of a gas bubble are its natural frequency and damping constant, which we have studied both experimentally and theoretically.

#### 3.1 Resonance frequency of gas bubbles in tubes

The results of this investigation are reported in the paper by Oğuz & Prosperetti (1998). The situations considered in that paper are shown in Fig. 16. The bubble is assumed to be spherical and enclosed in open or closed tubes. In that paper it is proven that, if  $f$  is the bubble natural frequency and  $f_0$  the natural frequency of a bubble with the same radius in an unbounded fluid, the following relation holds:

$$\left(\frac{f}{f_0}\right)^2 = \frac{a}{\langle \phi \rangle} \left\langle \frac{\partial \phi}{\partial r} \right\rangle, \quad (5)$$

where  $a$  is the bubble radius,  $\phi$  the velocity potential, and the angle brackets indicate an average over time and the surface of the bubble.

The Laplace equation for  $\phi$  was solved by a boundary integral method and the results compared with those of two simpler models. The first one, appropriate for relatively large bubbles, is one-dimensional and the bubble is simply a 'slice' of gas in the tube (see Fig. 18). With this model one finds, in place of (5),

$$\left(\frac{f}{f_0}\right)^2 = \frac{R^2}{4a} \left( \frac{1}{L_1^e} + \frac{1}{L_2^e} \right), \quad (6)$$

where  $R$  is the tube radius and  $L_{1,2}$  are suitably adjusted distances of bubble center from the ends of the tube. In the other model, more suitable for small bubbles, the bubble is approximated simply

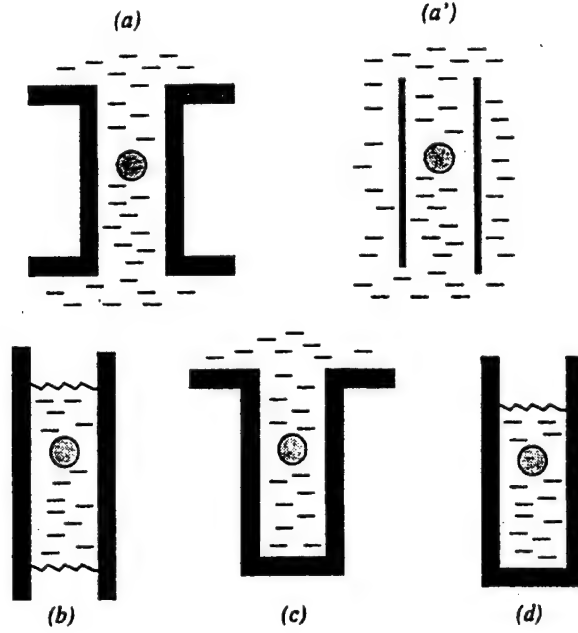


Figure 16: Various situations of a gas bubble in a tube.

by a monopole. Some typical results are shown in Fig. 17 and show that the approximate formula (6) is reasonably accurate; furthermore, it is evident from (6) that, if the dimensions  $R$ ,  $a$ ,  $L_{1,2}$  are not very different from each other, the natural frequency of the bubble is comparable to that of a bubble in an unbounded liquid.

### 3.2 Damping

The principal damping mechanisms affecting the oscillations of a gas bubble in a tube arise from viscosity and heat conduction. To account for the former in an approximate way we define an equivalent viscous damping rate  $\beta_v$  by

$$\mathcal{E}_v = 2b_v \int_0^{2\pi/\omega} \dot{z}^2 dt, \quad (7)$$

where  $\mathcal{E}_v$  is the energy dissipated by viscous effects during a cycle,  $\omega$  is the frequency of the oscillations, and  $z(t)$  the displacement of the liquid column bounding the bubble. By expressing the energy dissipated  $\mathcal{E}_v$  in terms of the standard viscous dissipation function of Fluid Mechanics, one finds

$$b_v = \frac{\omega}{2\pi} \mu L \int_0^{2\pi/\omega} dt \int_A dA \left[ \frac{\partial}{\partial r} \left( \frac{u}{V} \right) \right]^2. \quad (8)$$

Here the inner integral is over the cross section  $A$  of the tube,  $u$  is the local axial velocity of the liquid, and  $V$  is the mean velocity. It is possible to show that, at low frequency,

$$b_v = 4\pi m \frac{\mu}{\rho R^2}, \quad (9)$$

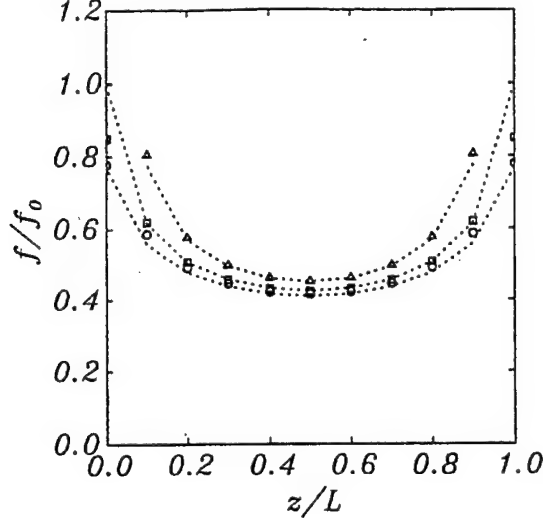


Figure 17: The natural frequency of a bubble of equilibrium radius  $a$  in a tube of radius  $R$  and length  $L$  as a function of the axial distance of the bubble center from the tube bottom for  $L/R = 10$ ,  $a/R = 0.5$ . The dotted lines are the results given by the approximate formula (6), while the symbols show the boundary integral results; circles: case (a) of Fig. 16; squares: case (a') of Fig. 16; triangles: case (b) of Fig. 16.

where  $m$  is the mass of the liquid column, while, at high frequencies,

$$b_v = m\sqrt{\mu\omega 2\rho R^2}. \quad (10)$$

Even though, due to the proximity of the tube walls, the viscous damping is more significant than in the case of a free bubble, in many cases (and in particular near resonance) it is thermal losses which dominate; these are analyzed in Chen & Prosperetti (1998) on the basis of the simplified model shown in Fig. 18.

In spite of the simplified geometry, the analysis is somewhat complicated and we refer the reader to the original paper for details. An example of the results is given in Fig. 19.

### 3.3 Experiments

A sketch of the experimental apparatus is shown in Fig. 20 (Geng et al. 1999). A drop of liquid was placed in the desired position in the upper part of a glass tube with diameters of 1 or 3 mm by means of a hypodermic syringe. A second drop was introduced so as to fill the lower part of the tube, leaving an air gap between its upper surface and the lower surface of the first drop. Most tests were run with water, but we also used silicon oil and a surfactant solution of 50 ppm Triton-X-100 in water. A stainless steel needle of suitable size, cut perpendicularly to the axis and plugged with silicon glue, was filed so as to fit snugly, and then inserted into the lower end of the tube leaving no air gap between its tip and the lower liquid region. The length of the liquid column separating the needle tip from the lower surface of the bubble was of the order of 10 - 20 mm. The needle was attached to a loudspeaker driven by an amplifier and function generator.

The objective of the experiment was to determine the amplitude of oscillation of the gas volume as a function of frequency. For this purpose a CCD camera was used to take digital snapshots of



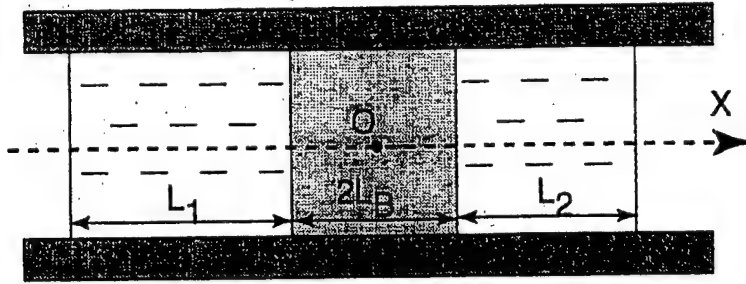


Figure 18: Simplified configuration for the calculation of thermal losses in the course of gas bubble oscillations.

the bubble at equally spaced time intervals. Sample sequences of the pictures acquired are shown in Fig. 21.

The digital images produced by the CCD camera were scanned to determine the position of the surface bounding the gas space. From this digitized version of the bubble surface, with the assumption of axial symmetry, the instantaneous bubble volume could be computed. The image processing was carried out automatically in real time during the course of the experiment thus avoiding the need to store the images.

For all frequencies we took 200 data points and least-squares fitted them to an expression of the form

$$L(t) = A_B \cos(\omega t + \phi) + ct + L', \quad (11)$$

where  $\omega = 2\pi f$  is the angular frequency of the driver and  $A_B$ ,  $\phi$ ,  $c$ , and  $L'$  are determined from the fit. The largest difference between the value of  $L'$  obtained from the fit and the initial value  $L_0$  was about 1%. The term  $ct$  was included to account for any drift due, e.g., to loss of liquid leaking past the needle, but in all cases it was found to be very small with a contribution of the order of 1% or less.

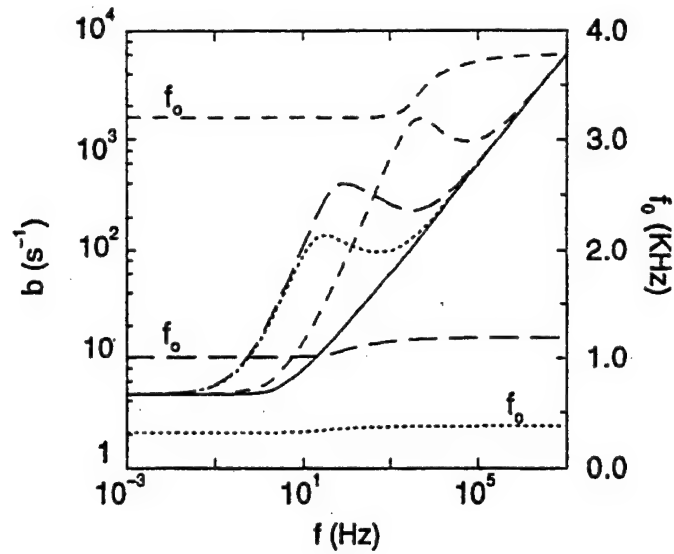


Figure 19: Natural frequency (right scale) and total damping constant (left scale) for the driven oscillations of an air bubble in a 1 mm-radius tube as functions of the driving frequency  $f$ . The tube is 10 mm long and the bubble is located at the center. The solid line is the viscous contribution to the damping. The dotted lines are for  $2L_B = 10$  mm, the long dashes for  $2L_B = 0.1$  mm, and the short dashes for  $2L_B = 0.1$  mm. The liquid is water, the temperature 20 °C, and the undisturbed pressure atmospheric.

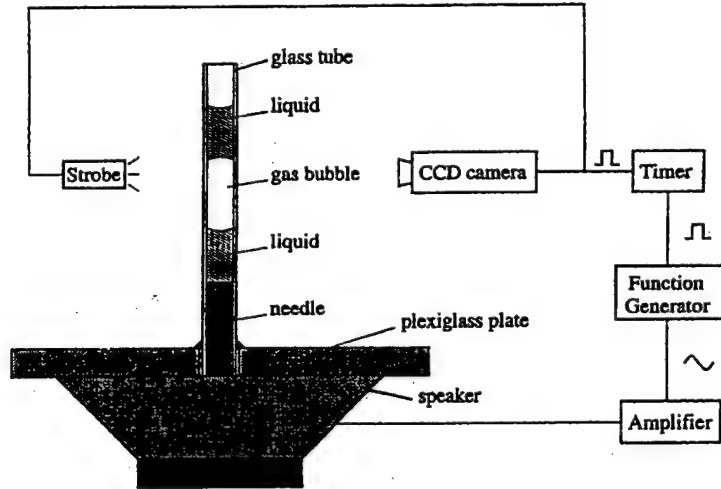


Figure 20: Sketch of the experimental apparatus for the study of the forced oscillations of a gas bubble in a tube. The tube containing the two liquid columns separated by the air bubble is inserted onto a needle fastened to a loudspeaker cone. The loudspeaker is driven by a function generator connected through an amplifier.

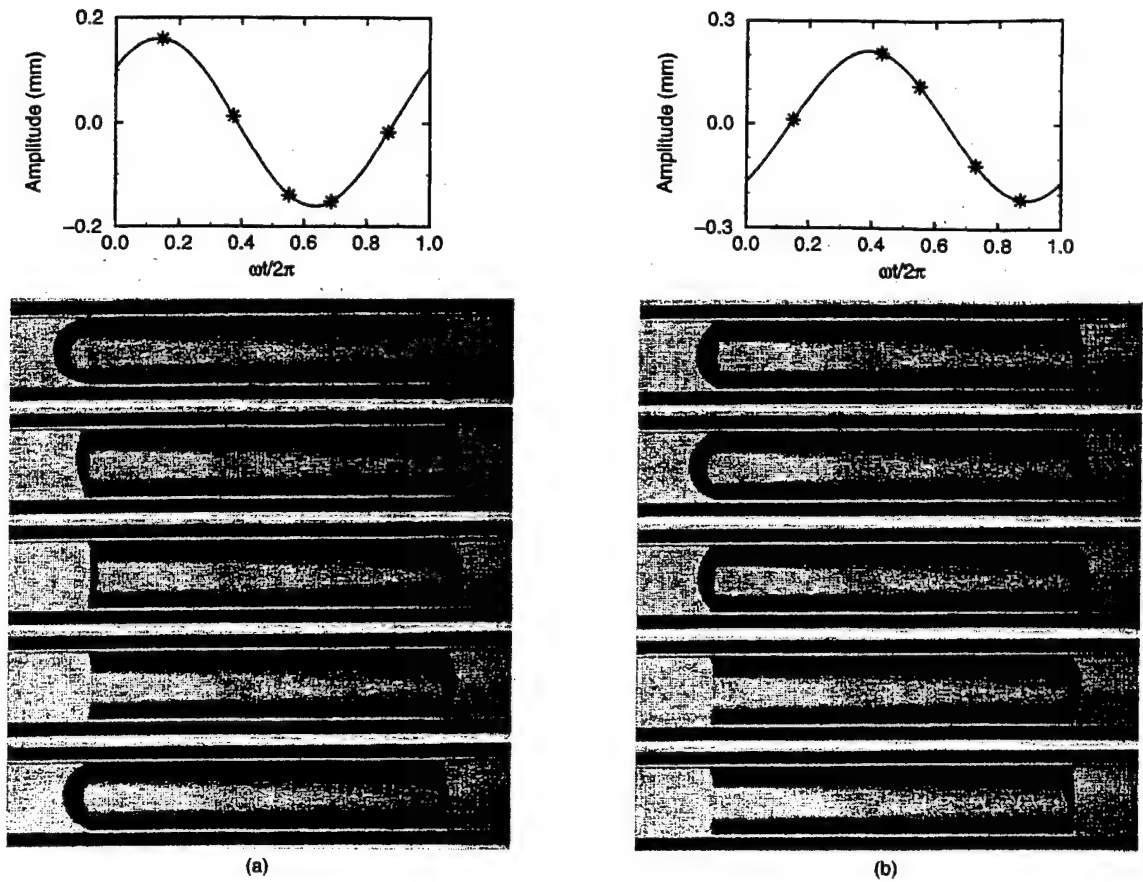


Figure 21: Sample sequences of gas bubble oscillations at 150 Hz (left) and 200 Hz (right) for pure water in the 1 mm-diameter tube. The bubble resonance frequency is 173 Hz. The graphs above the photographic sequences show the position of each frame in the oscillation cycle. Time  $t = 0$  corresponds to the maximum upward displacement of the needle. Gravity acts from left to right. The needle is outside the frame to the right. The position of the contact line is clearly visible. Note the small drops on the glass surface indicating that it is not covered by a liquid film. The bright bands along the sides of the tube are due to refraction in the glass.

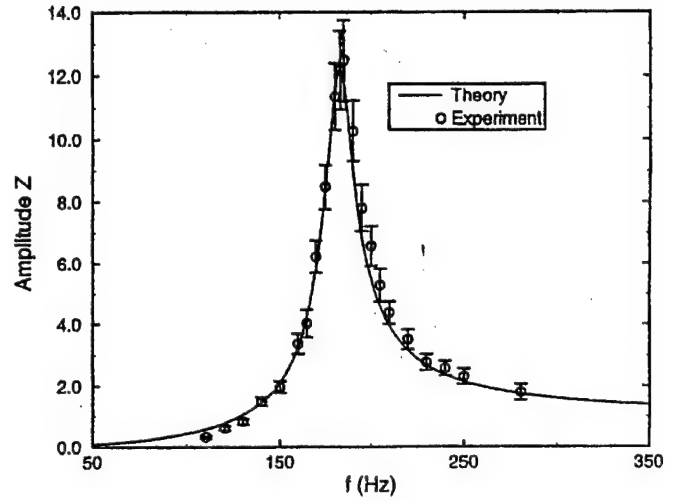
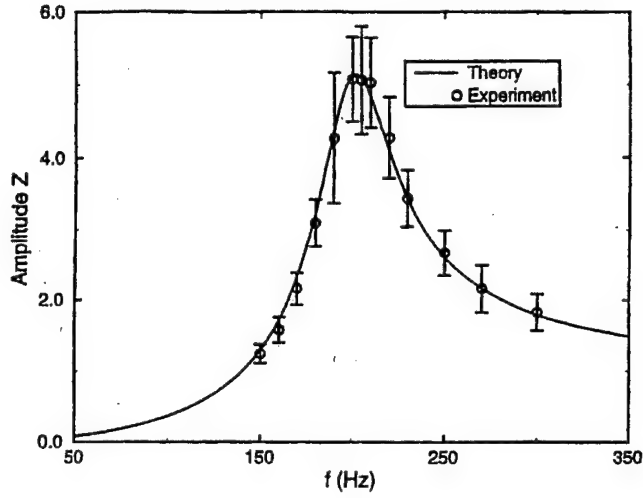


Figure 22: (left) Comparison between theory (solid line) and experiment for pure water in the 1 mm-tube. The normalized amplitude  $Z$  is the oscillation amplitude  $A_B$  of (11) divided by the analogous quantity for the metal needle driving the oscillations.

Figure 23: (right) Comparison between theory (solid line) and experiment for pure water in the 3 mm-tube. The normalized amplitude  $Z$  is the oscillation amplitude  $A_B$  of (11) divided by the analogous quantity for the metal needle driving the oscillations.

A comparison of theory and experiment for two cases is shown in Fig. 22 for water in tubes with 1 and 3 mm diameter. The agreement is very good and substantiates the analysis. It should be noted that, if thermal effects were neglected, the resonance peak would be over 3 times as high. It is thus seen that, due to thermal losses, the  $Q$  of the resonance of a gas bubble in a tube is not very large.

# The natural frequency of oscillation of gas bubbles in tubes

H. N. Oğuz and A. Prosperetti

Department of Mechanical Engineering, The Johns Hopkins University, Baltimore, Maryland 21218

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A numerical study is presented of the natural frequency of the volume oscillations of gas bubbles in a liquid contained in a finite-length tube, when the bubble is not small with respect to the tube diameter. Tubes rigidly terminated at one end, or open at both ends, are considered. The open ends may be open to the atmosphere or in contact with a large mass of liquid. The numerical results are compared with a simple approximation in which the bubble consists of a cylindrical mass of gas filling up the cross section of the tube. It is found that this approximation is very good except when the bubble radius is much smaller than that of the tube. An alternative approximate solution is developed for this case. The viscous energy dissipation in the tube is also estimated and found generally small compared with the thermal damping of the bubble. This work is motivated by the possibility of using gas bubbles as actuators in fluid-handling microdevices. © 1998 Acoustical Society of America. [S0001-4966(98)02606-X]

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## INTRODUCTION

An extensive literature exists on the small-amplitude volume oscillations of gas bubbles in unbounded liquids, near rigid plane boundaries and free surfaces (see, e.g., Strasberg, 1953; Howkins, 1965; Blue, 1966; Plesset and Prosperetti, 1977; Apfel, 1981; Scott, 1981; Prosperetti *et al.*, 1988; Oğuz and Prosperetti, 1990; Prosperetti, 1991). The case of bubbles confined in channels and tubes, however, does not seem to have been considered before except in a brief unpublished report by Devin (1961). Of course, when the radius of the tube is much larger than the bubble—as would be the case, for example, for bubbles entrained in ordinary macroscopic flows—the results for bubbles in unbounded liquids can be used to a good approximation. In the situations of concern here, however, the size of the bubble is not small and the effect of the proximity of the boundary very significant.

The situations considered in this paper are all axisymmetric and are sketched in Fig. 1. The bubble is inside a liquid-filled, finite-length, rigid-walled tube that may be open at both ends [Fig. 1(a), (a'), and (b)], or rigidly terminated at one end and open at the other [Fig. 1(c) and (d)]. The open end(s) of the tube may be in contact with the atmosphere [Fig. 1(b) and (d)], or with a large mass of the same liquid [Fig. 1(a), (a'), and (c)].

Our interest in these problems is motivated by the possibility to use gas bubbles as actuators in the small fluid-handling systems that advances in silicon manufacturing technology are rendering possible (see, e.g., Fujita and Gabriel, 1991; Lin *et al.*, 1991; Gravesen *et al.*, 1993). These include bioassay chips, integrated micro-dosing systems, miniaturized chemical analysis systems, and others. The advantage of bubbles in this setting would be the possibility to power them remotely by ultrasonic beams with no need for direct contact between the actuator and the power supply. A particularly intriguing possibility in this regard may be offered by the ability of ultrasound to propagate through living tissue.

While the scale that we envisage is of the order of one millimeter or less and the flow velocities relatively small, so that viscous effects would not be negligible, it seems natural for a first analysis of this problem to start from a consideration of the inviscid case, treating viscous effects in an approximate way (see Sec. IV). The attending simplification enables us to focus with greater clarity on the inertial aspects of the bubble-fluid interaction, which are one of the dominant aspects of the system. Second, it will be easier to establish a connection with the available results for the unbounded case. Third, one can envisage situations in which viscosity is indeed negligible, such as an oscillation frequency so large that the viscous boundary layer is much thinner than the tube.

Since, in order to maximize the effectiveness of the actuator, it is desirable to operate near resonance conditions, the natural frequency of the bubble is the most significant quantity to be determined. This is the objective of this paper. In the future we shall consider forced oscillations, damping mechanisms, and nonlinear effects.

## I. FORMULATION

As shown by Strasberg (1953; see also Oğuz and Prosperetti, 1990), it is possible to obtain a relation for the natural frequency directly by using the analogy with the capacitance problem of electrostatics. To this end we start from the condition expressing the balance of normal forces at the bubble surface:

$$p_i = p_L + \sigma \mathcal{C}. \quad (1)$$

Here,  $p_i$  is the pressure in the bubble, assumed spatially uniform,  $p_L$  the pressure in the liquid at the bubble surface,  $\sigma$  the surface tension coefficient, and  $\mathcal{C}$  the local curvature of the interface. Upon using the (linearized) Bernoulli integral to express  $p_L$  in terms of the velocity potential  $\phi$  and the

# Thermal processes in the oscillations of gas bubbles in tubes

X. M. Chen and A. Prosperetti

*Department of Mechanical Engineering, The Johns Hopkins University, Baltimore, Maryland 21218*

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The forced oscillations of a system consisting of two finite liquid columns in a duct separated by a gas bubble are studied in the linear approximation. It is found that thermal processes in the gas induce a very significant damping in the system, which can exceed viscous damping even in capillaries with a submillimeter diameter. The study is motivated by the possibility of using gas bubbles as actuators in microdevices. © 1998 Acoustical Society of America.

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## INTRODUCTION

It is well known that a gas bubble pulsating in a large liquid mass loses energy by viscosity, acoustic radiation, and thermal conduction (see, e.g., Apfel, 1981; Prosperetti, 1984, 1991). The additional dissipation due to gas diffusion is essentially always negligible, while thermal effects due to phase change are insignificant for a liquid like water at normal temperature. Over a wide range of bubble radii and frequencies, thermal conduction is by far the most significant dissipative mechanism. In addition to energy dissipation, thermal processes also influence the stiffness of the bubble the behavior of which, in general, will be intermediate between isothermal and adiabatic. The corresponding processes for bubbles pulsating in a duct have not been studied, and they constitute the object of this paper.

The motivation for this work lies in the possibility to use pulsating bubbles as actuators in small fluid-handling devices such as those made possible by recent progress in silicon manufacturing techniques (see e.g., Fujita and Gabriel, 1991; Lin and Pisano, 1991; Gravesen *et al.*, 1993). Of course, energy dissipation is important as it determines the width of the resonance and the magnitude of the response under forced oscillation. The stiffness of the bubble determines the resonance frequency.

## I. FORMULATION

Since this is the first study devoted to the problem, we feel justified in introducing some approximations that will, on the one hand, simplify the analysis and, on the other, facilitate a comparison with the established results for a spherical bubble.

In the first place, and just as in the case of a spherical bubble, we assume the wavelength in the gas to be much larger than both the lateral dimensions of the channel and the axial length of the bubble, so that the gas pressure can be considered spatially uniform and only a function of time (see, e.g., Prosperetti, 1991). Another approximation in common with the standard analyses for spherical bubbles is the neglect of the vapor contribution to the bubble internal pressure and of phase change processes, both approximations being motivated by the consideration of only relatively "cold" liquids (see, e.g., Plesset and Prosperetti, 1977).

In the case of bubbles in large liquid masses a substantial simplification arises from the assumption of spherical shape. Here we introduce a parallel assumption on the shape of the bubble: since our interest lies in channels with a diameter of the order of 1 mm or less, we take the bubble to occupy an entire section of the channel, ignoring the problems associated with contact angles and the detailed shape of the gas-liquid interface (Fig. 1). The gas volume is therefore assumed to be bounded by two flat liquid surfaces orthogonal to the axis of the channel, and by the surface of the channel comprised between these two surfaces. The amplitude of the oscillations is taken to be so small that the problem can be linearized, and the liquid surfaces bounding the gas volume are supposed to move remaining flat and orthogonal to the channel walls; the complexities associated with the motion of the gas-liquid-solid contact line are therefore ignored. This approximation has the consequence of rendering it impossible to account for a velocity profile in the liquid. Energy dissipation due to liquid viscosity will be reintroduced in an approximate way later (Sec. V). In spite of the relative crudeness of this model, one may expect the results to be a valid first estimate of the quantitative effects of the physical processes involved.

Let  $x_1(t)$  and  $x_2(t)$  denote the time-dependent positions of the two gas-liquid interfaces, both measured from the midpoint of the undisturbed bubble, and define

$$x_2 - x_1 = 2L_B[1 + X(t)], \quad (1)$$

where  $2L_B$  is the undisturbed length of the bubble. We shall only consider the steady-state problem in which the time dependence of all disturbances is proportional to  $\exp i\omega t$ .

Due to the translational invariance of the channel and to the fact that, in a linear problem, the perturbation of any quantity is proportional to that of any other, one may write without loss of generality the gas pressure in the form

$$p(t) = p_0[1 - \Phi X(t)], \quad (2)$$

where  $p_0$  is the equilibrium value and  $\Phi$  is a complex constant. It will be noted that this relation may be considered as the linearization of a relation of the type  $pV^\Phi = \text{const}$  (where  $V$  is the bubble volume), so that  $\Phi$  may be regarded as a (complex, frequency dependent) polytropic index.

# The oscillation of gas bubbles in tubes: Experimental results

X. Geng, H. Yuan, H. N. Oğuz, and A. Prosperetti<sup>a)</sup>

Department of Mechanical Engineering, The Johns Hopkins University, Baltimore, Maryland 21218

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An experimental study is presented of the frequency dependence and damping of the forced volume oscillations of gas bubbles in liquid-filled tubes. The bubbles occupy the entire section of the tube and are driven by a needle attached to a loudspeaker cone. The liquids used were water, a water-surfactant solution, and silicon oil, and the tube diameters were 1 and 3 mm. The results are in excellent agreement with the theory developed in two earlier papers. This work is motivated by the possibility of using gas bubbles as actuators in fluid-handling microdevices. © 1999 Acoustical Society of America. [S0001-4966(99)04807-9]

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## INTRODUCTION

In two recent papers, a theory was presented for the natural frequency<sup>1</sup> and damping<sup>2</sup> of the oscillations of gas bubbles in liquid-filled tubes. In the present study, we describe an experimental investigation that is in excellent agreement with the theoretical predictions.

While an extensive literature exists on the small-amplitude volume oscillations of gas bubbles in unbounded liquids (reviewed, for instance, in Refs. 3–6), the physics of bubbles confined in channels and tubes presents such substantial differences that a specific investigation is necessary. This statement is especially applicable to the bubbles considered here, which are large enough to occupy an entire section of the tube. Indeed, in this case, the inertia of the system is entirely dependent on the length of the liquid columns on either side of the bubble, rather than being proportional to the bubble volume. Furthermore, the liquid flow in the tube can be strongly affected by viscosity which, for free bubbles, is usually unimportant except for very small radii.

This work is motivated by the possibility to use gas bubbles as actuators in the small fluid-handling systems that advances in silicon manufacturing technology render possible (see, e.g., Refs. 7–9). One of the attractive features of such an application is that gas bubbles can be powered remotely by ultrasonic sources with no need for direct contact between the actuator and the power supply.

## I. EXPERIMENT

Glass tubes with a diameter of 1 and 3 mm were used with water, silicon oil, and a surfactant solution of 50 ppm Triton-X-100 in water. This amount is about 37% of the critical micellar concentration. A drop of liquid was placed in the desired position in the upper part of the tube by means of a hypodermic syringe. A second drop was introduced so as to fill the lower part of the tube, leaving an air gap between its upper surface and the lower surface of the first drop. A stainless-steel needle of suitable size, cut perpendicularly to the axis and plugged with silicon glue, was filed so as to fit snugly, and then inserted into the lower end of the

tube leaving no air gap between its tip and the lower liquid region. The length of the liquid column separating the needle tip from the lower surface of the bubble was of the order of 10–20 mm. The needle was attached to a loudspeaker driven by an amplifier and function generator while the glass tube was mounted in a fixed plexiglass plate (Fig. 1).

The objective of the experiment was to determine the amplitude of oscillation of the gas volume as a function of frequency. For this purpose, a charge-coupled device (CCD) camera was used to take digital snapshots of the bubble at equally spaced time intervals. Since the speed of the camera was not sufficient to record the necessary number of pictures during a single cycle, pictures were taken at time intervals  $nT + \Delta t$ , with  $T$  the period of the needle oscillations,  $n$  a suitable integer (such that  $nT = 500$  ms), and  $\Delta t = 0.1$  ms, which gives 30 to 100 frames per cycle depending on the driving frequency. Sample sequences of the pictures acquired in this way are shown in Figs. 2–5.

The digital images produced by the CCD camera were scanned to determine the position of the surface bounding the gas space. From this digitized version of the bubble surface, with the assumption of axial symmetry, the instantaneous bubble volume  $V(t)$  could be computed. The image processing was carried out automatically in real time during

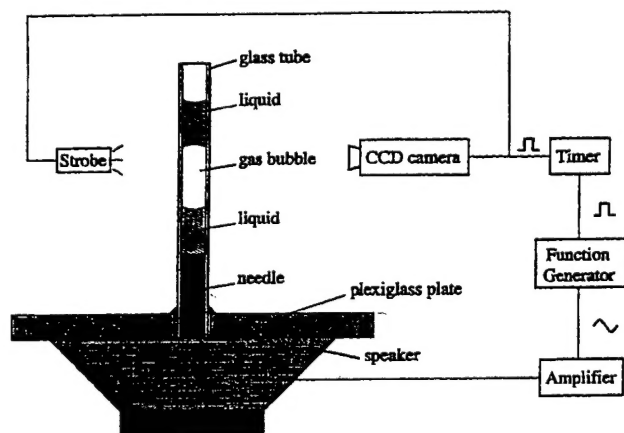


FIG. 1. Sketch of the experimental apparatus. The tube containing the two liquid columns separated by the air bubble is inserted onto a needle fastened to a loudspeaker cone. The loudspeaker is driven by a function generator connected through an amplifier.

<sup>a)</sup>Electronic mail: prosper@titan.me.jhu.edu





# Growth and collapse of a vapor bubble in a small tube

H. Yuan, H.N. Oğuz, A. Prosperetti\*

*Department of Mechanical Engineering, The Johns Hopkins University, Baltimore, MD 21218, USA*

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## Abstract

A model for the description of the growth and collapse of a vapor bubble in a small tube is presented and some typical results are illustrated. It is found that the maximum volume of the bubble and its lifetime depend in a complex way on the channel geometry and the initial energy distribution. However, during most of the bubble's lifetime, the internal pressure is very small and the dynamics mostly governed by the external pressure. The motivation for this work is offered by the possibility to use vapor bubbles as actuators in fluid-handling microdevices. © 1999 Elsevier Science Ltd. All rights reserved.

## 1. Introduction

The silicon microfabrication techniques recently developed render new technologies possible and novel regions of parameter space worthy of study. An example is the use of gas or vapor bubbles as actuators without mechanical moving parts. An intriguing possibility explored in Ref. [1] is a micropump based either on a Marangoni effect or on the suitably phased growth and collapse of bubbles generated by small heaters in liquid-filled microchannels.

The device described in Ref. [1] operates at frequencies of the order of 1 Hz in tubes with a diameter of a few micrometers. While that size range may be useful for some applications, here we are interested in tubes with a diameter of the order of a hundred micrometers that are of interest, for example, for drug delivery, on-chip chemical analysis, and others [2–5]. The exploration of vapor bubble formation in this size range is in a very early stage. A limited number of papers [6,7] deal with boiling in confined spaces and narrow gaps

[8]. Models of bubbles growing in pores and pore networks are encountered in the literature on boiling in porous media [9,10], but in all these situations heat transfer to the bubble occurs from the entire solid surface rather than from a very localized heated region as in the situation of present concern. A similar comment applies to papers motivated by phase-change phenomena in micro heat pipes [11,12].

Closer to the situation investigated here is the modeling of drop ejection in ink-jet printers [13,14], and of bubble growth on microheaters [1,15,16]. In order to deal with the rather formidable difficulties of the problem, these models contain several idealizations that we try to improve upon in the present paper. We allow for the presence of more than one spatial dimension in the problem, for the momentum of the liquid, the presence of the tube wall, and several other effects. While the present model can in no way claim to be definitive, it is less idealized than others and sheds an interesting light on several of the controlling aspects of the phenomenon. For example, one of the interesting conclusions of the study is the extreme brevity of the time during which the bubble internal pressure is large. During most of the bubble life, the internal pressure is

\* Corresponding author.



# The pumping effect of growing and collapsing bubbles in a tube

H Yuan and A Prosperetti†

Department of Mechanical Engineering, The Johns Hopkins University, Baltimore, MD 21218, USA

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**Abstract.** The flow induced by the suitably timed growth and collapse of one or several bubbles in a finite tube joining two liquid reservoirs is simulated by a simple quasi-one-dimensional model. Viscous and surface tension effects are accounted for in an approximate manner. It is shown that, in certain parameter ranges, the system is capable of a net pumping action that moves the liquid from one reservoir to the other even in the presence of an adverse pressure difference. The fact that this net pumping effect is also encountered in the case of a single bubble, provided it is not located at the midpoint of the tube, is particularly remarkable. The mechanism responsible for this result is discussed. In practice the effect can be exploited to build a micropump by embedding electrical heaters in the wall of a small channel.

## 1. Introduction

The potential of phase change phenomena to achieve actuation in small systems has attracted the attention of several investigators. The earliest practical application is the ink-jet printer (Allen *et al* 1985, Asai 1989, 1991, Burr *et al* 1996, Chikanawa *et al* 1996), but other applications have also been described (Ji *et al* 1991, Lin *et al* 1991, Bergstrom *et al* 1995). In particular, Jun and Kim (1996) have described a peristaltic micropump based on the sequential growth of vapor bubbles along a small channel with embedded wall heaters. The very low repetition rate achieved in this work, about 1 s, makes the device impractical for most applications. This characteristic time cannot be shortened by the several orders of magnitude necessary to make it useful without a deeper understanding of the thermal and dynamic aspects of the system. More generally, the widespread application of bubbles in practical devices requires a better understanding of much basic physics that is, at present, still lacking.

Although a great deal is known about bubble dynamics in unconfined volumes (see e.g. Plesset and Prosperetti 1977, Feng and Leal 1997), little work has been carried out on bubbles confined in narrow spaces with characteristic dimensions comparable with their size. Most of the available literature has been motivated by the development of compact heat exchangers and deals with boiling heat transfer in confined spaces (see e.g. Borowsky and Sernas 1994, Kew and Cornwell 1994, Peng and Wang 1994). The situations in which bubbles would be used in micro-systems, however, are very different. Vapor bubbles would be generated by applying short current pulses to micro-heaters, while the rest of the system would remain at ambient temperature. In

addition to Asai's work, whose situation is quite different from the one treated here, the only study addressing such conditions is our own recent one (Yuan *et al* 1999), in which we coupled an inviscid boundary integral formulation for the fluid mechanic aspects of the problem with a simplified treatment of the heat transfer and phase change.

In spite of the many simplifications adopted, that model still retains a high degree of complexity and does not lend itself easily to the simulation of the repeated action of many bubbles growing and collapsing in concert that lies at the heart of the peristaltic pumping action considered in this paper. For this reason, here we develop and study a simpler model which, nevertheless, incorporates many of the basic aspects of the actual physical situation.

## 2. The model

In this study we are interested in exploring under what conditions a sequence of growth and collapse cycles of  $N$  bubbles suitably positioned in space and time can generate an average unidirectional flow along a tube connecting two reservoirs at different pressures. In practice the bubbles would be generated inside a small liquid-filled tube, on electric heaters embedded in the tube wall. In order for the bubble generation to be precisely repeatable, it is necessary to rely on homogeneous or nearly-homogeneous nucleation and the heater surface must be brought to a high temperature. The duration of this phase must be kept very short in order both to avoid damaging the heaters and to input as little heat as possible into the system. In our earlier paper (Yuan *et al* 1999) we found that, in the course of the expansion, the bubble internal pressure quickly falls to near the saturation value at the undisturbed liquid temperature due to the work of expansion and cooling of the surrounding fluid and solid

† Also Department of Applied Physics, Twente Institute of Mechanics, and Burgerscentrum, University of Twente, Enschede, The Netherlands.

## ARTICLES

## Growth and collapse of a vapor bubble in a narrow tube

E. Ory, H. Yuan, and A. Prosperetti<sup>a)</sup>*Department of Mechanical Engineering, The Johns Hopkins University, Baltimore, Maryland 21218*

S. Popinet and S. Zaleski

*Laboratoire de Modélisation en Mécanique UMR-CNRS 7607, Université Pierre et Marie Curie,  
4 Place Jussieu, 75252 Paris Cedex 05, France*

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The fluid mechanical aspects of the axisymmetric growth and collapse of a bubble in a narrow tube filled with a viscous liquid are studied numerically. The tube is open at both ends and connects two liquid reservoirs at constant pressure. The bubble is initially a small sphere and growth is triggered by a large internal pressure applied for a short time. After this initial phase, the motion proceeds by inertia. This model simulates the effect of an intense, localized, brief heating of the liquid which leads to the nucleation and growth of a bubble. The dimensionless parameters governing the problem are discussed and their effects illustrated with several examples. It is also shown that, when the bubble is not located at the midpoint of the tube, a net flow develops capable of pumping fluid from one reservoir to the other. The motivation for this work is offered by the possibility to use vapor bubbles as actuators in fluid-handling microdevices. © 2000 American Institute of Physics. [S1070-6631(00)01906-1]

## I. INTRODUCTION

In an ink-jet printer drops are produced and ejected due to the rapid growth of bubbles in a narrow flow passage just upstream of the nozzle exit (see, e.g., Refs. 1–3). This is but one example of the possible application of vapor bubbles in small devices, where they offer the interesting potential of mechanical actuation, pumping, and other flow effects without mechanical moving parts. In these situations bubbles are produced by small heaters in the tube wall to which a brief current pulse is applied. A very small mass of liquid reaches a temperature comparable to the critical temperature, and a vapor bubble nucleates and rapidly grows. In a recent paper we have studied the thermal and attendant phase-change processes that take place in such a situation.<sup>4</sup> We found that, due to the work of expansion and vapor condensation on the cold liquid surface, the initial high vapor pressure in the bubble is very quickly dissipated. The rest of the bubble evolution essentially proceeds by inertia. In that study the fluid mechanical aspects of the problem were treated by means of a simplified model based on potential flow and an approximate treatment of viscous effects. The purpose of this article is to simulate the actual flow behavior by solving numerically the full Navier–Stokes equations. We shall however simplify the thermal problem by assuming that the bubble internal pressure at time 0 is briefly raised to a level  $\Delta p$  above the saturation level  $p_0 \approx 0$  corresponding to the undisturbed liquid

temperature. This high internal overpressure is maintained for a time  $\tau$ , after which it falls to zero again. In another recent paper<sup>5</sup> we have compared the results of this simple model with the one of Ref. 4 finding a very good agreement.

The advancing of a gas bubble in a tube is a classic problem treated by many authors since the early work of Bretherton;<sup>6</sup> Ref. 7 gives a good review and some more recent work can be found in Refs. 8 and 9. The situation considered in this paper is however different as, first, the bubble undergoes a significant volume change and, second, the flow is highly transient as opposed to the steady or quasisteady situations considered before. In addition, inertial effects are significant and therefore we deal with the full Navier–Stokes equations, rather than the Stokes equations as did many of the previous authors. The situation considered in the study of boiling in porous media (see, e.g., Ref. 10–13) is also quite different from the present one as, in that case, heat is added to the system continuously from the entire pore wall, rather than locally and impulsively as here. Among the earlier work on the specific problem studied here, the closest one is that of Asai *et al.*<sup>1</sup> who, however, developed a quasi-one-dimensional model and were chiefly concerned with drop formation in the ink-jet system rather than with specific fluid mechanical phenomena.

An interesting and unexpected result that we find is that a single bubble cyclically growing and collapsing away from the midpoint of the channel is capable of inducing a mean unidirectional flow capable of pumping liquid from one reservoir to the other.

<sup>a)</sup>Also at: Department of Applied Physics, Twente Institute of Mechanics, and Burgerscentrum, University of Twente, AE 7500 Enschede, The Netherlands.